Polarization lifetime near an intrinsic depolarizing resonance

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We have measured the time dependence of the polarization of a stored proton beam near an intrinsic depolarizing resonance. The distance to the resonance was varied by changing the vertical tune of the storage ring. The measurements are consistent with exponential decay and with a simple model for the change of the lifetime as the resonance is approached. It was originally expected that the presence of an internal target stimulates depolarization; however, we found no evidence for such an effect. [S1063-651X(97)00109-8]

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I. INTRODUCTION

When a polarized proton beam first became available in the Cooler Ring at the Indiana University Cyclotron Facility (IUCF), it was important to settle the question of whether electron cooling has an adverse effect on the beam polarization. This was found not to be the case, as predicted [1]. In fact, the first measurement of a polarization lifetime with a 200 MeV proton beam [2] yielded a value in excess of 6 h, indicating that polarization in a storage ring is remarkably persistent. In later Cooler experiments a small but significant decrease of the polarization with time was sometimes observed. It was conjectured that the presence of an internal target is responsible.

In this report we explore the polarization lifetime of a cooled beam in the neighborhood of an intrinsic resonance with and without an internal target. In Sec. II we discuss theoretical aspects of polarization in storage rings. Section III contains technical details related to the stored beam and the polarimeter used. The polarization lifetime measurements are presented in Sec. IV, followed by a discussion in Sec. V.

II. BEAM DEPOLARIZATION BY DEPOLARIZING RESONANCES

A. Spin motion

The magnetic moment of an orbiting particle precesses around the encountered magnetic fields. We limit this discussion to spin-$\frac{1}{2}$ particles. In this case, the spin can be described by a three-component vector **S**, parallel to the magnetic moment. In general, the motion of **S** in one revolution, starting and ending at a given point z on the trajectory, is a rotation around a vector $\hat{n}_{\mathrm{CO}}(z)$, called the "spin closed orbit." The component of the spin vector parallel to $\hat{n}_{\mathrm{CO}}$, $\mathbf{S}_{\parallel}$, is preserved, while the component $\mathbf{S}_{\perp}$ orthogonal to $\hat{n}_{\mathrm{CO}}$ rotates around it with a frequency $f_r \nu_s$, where f_r is the beam revolution frequency and ν_s is called the "spin tune." In a machine with only vertical fields, the spin closed orbit is vertical independent of z (except near a resonance, see below). In this case, the spin tune equals $\nu_s = G\gamma$, where G is the anomalous magnetic moment (for protons, $G = 1.792\,847$), and γ is the relativistic Lorentz factor. Nonvertical fields due to focusing elements or field imperfections become important when they are encountered in resonance with the spin tune, giving rise to "intrinsic" resonances when $G\gamma = m \pm \nu_{yR}$ (m is an integer and ν_{yR} is the vertical betatron tune of the ring), or to "imperfection" resonances when $G\gamma = n$ (n is an integer). Here, we ignore complications which might arise from coupling of the vertical to the horizontal and longitudinal (synchrotron) particle motions.

The strength Γ of an intrinsic resonance depends on the optics of the ring lattice and is proportional to the betatron amplitude of the particle. The "distance" δ of the resonance is given by the difference between the actual tune ν_y and the tune for which the resonance condition is met, or $\delta = \nu_{yR} - \nu_y$.

Near a resonance, the spin closed orbit $\hat{n}_{\mathrm{CO}}$ deviates from the vertical direction $\hat{y}$ by an angle α (as shown in Fig. 1) and rotates around $\hat{y}$ as a function of z, and, for a fixed z, as a function of time. The angle α is given by the resonance strength Γ and the distance δ from the resonance by $\Gamma = \delta \tan\alpha$ [3]. The vertical, stable component of the spin vector near a resonance is therefore $\mathbf{S}_{\parallel}\cos\alpha$. On resonance ($\delta=0$) the vertical component of $\hat{n}_{\mathrm{CO}}$ is zero. Since the resonance condition depends on beam energy, a resonance can be crossed during acceleration or deceleration. An intrinsic resonance can also be crossed by changing the betatron tune. In the process of crossing a resonance, the spin closed orbit starts out pointing up (for instance), becomes horizontal exactly on resonance, and ends up pointing down. If the crossing is slow, the spin vector $\mathbf{S}_{\parallel}$ follows the spin closed orbit, and changes sign.

B. Depolarization

In the preceding section we have described the spin motion of a single particle. An ensemble of particles would coherently follow this motion while the magnitude of its polarization is preserved. Consequently, in order to explain *beam depolarization as a function of time*, a mechanism is needed that causes the ensemble of spin vectors to *decohere*.

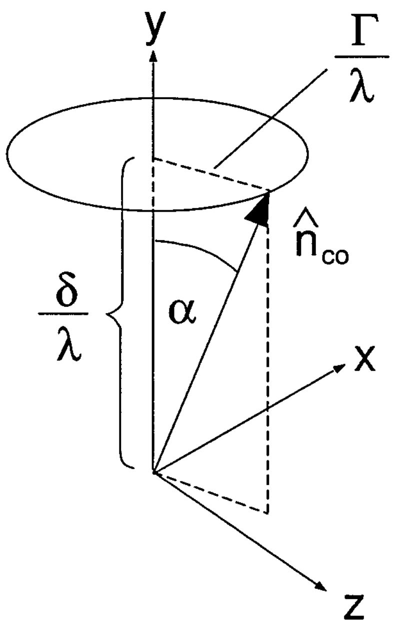

FIG. 1. The spin closed orbit $\hat{n}_{\rm CO}$ for a particle at a distance δ from an intrinsic resonance of strength Γ. The parameter λ is given by $\lambda^2=\delta^2+\Gamma^2$.

In a real machine with a beam of finite emittance, the betatron amplitudes a and tunes ν of individual particles are distributed around their mean values with respective variances σ_a and σ_ν. Thus each particle in the beam has its individual value for the strength Γ and distance δ from the resonance.

Let us now assume that there is a nonconservative process operating which mixes the phase space of the stored beam. Such a process could, for instance, induce a sudden change $\Delta\nu_y$ of the vertical tune ν_y of a given beam particle. This changes the distance to the resonance by $\Delta\delta=\Delta\nu_y$. This in turn changes the angle α of the spin closed orbit by $\Delta\alpha=|d\alpha/d\delta|\Delta\delta=[\Gamma/(\Gamma^2+\delta^2)]\Delta\nu_y$. Since the stable direction of the spin vector is now the component along the new spin closed orbit, the polarization decreases by $\Delta P/P=1-\cos\Delta\alpha\cong\Delta\alpha^2/2$. The frequency ξ with which the postulated tune changes occur depends on the (unknown) mechanism. The mean square value of the size of the changes, $\langle\Delta\nu_y^2\rangle$, depends on the tune distribution which determines the probability for finding the tune values before and after the change (for instance, for a hypothetical distribution that allows only the values ν_1 and ν_2, the only possible change would be $|\nu_1-\nu_2|$). Combining the above, we arrive at the following expression for the polarization lifetime τ_P:

$$\frac{1}{\tau_P}\equiv-\frac{1}{P}\frac{dP}{dt}=\frac{\xi}{2}\langle\Delta\nu_y^2\rangle\left(\frac{\Gamma}{\Gamma^2+\delta^2}\right)^2. \quad (1)$$

Equation (1) offers no insight into the *mechanism* that mixes the distribution in tune space, other than that it somehow affects δ, the distance to the resonance of individual particles. However, Eq. (1) does tell us how the polarization lifetime decreases as the resonance is approached. Rewriting Eq. (1) yields

$$\tau_P(\nu_y)=C_\delta[(\nu_y-\nu_{yR})^2+\Gamma^2]^2/\Gamma^2, \quad (2)$$

where ν_y and ν_{yR} are the actual tune and the tune at which resonance would be encountered, Γ is the resonance strength, and C_δ is an unknown constant.

It is also conceivable that the mixing occurs in transverse phase space. This would cause changes in the betatron amplitude of individual particles, and thus in their value for the resonance strength Γ. The derivation of the corresponding expression for the polarization lifetime is analogous, except that the derivative $d\alpha/d\Gamma$ occurs, instead of $d\alpha/d\delta$, and

$$\tau_P(\nu_y)=C_\Gamma[(\nu_y-\nu_{yR})^2+\Gamma^2]^2/(\nu_y-\nu_{yR})^2. \quad (3)$$

In order to apply Eqs. (2) and (3) to an ensemble of beam particles, one has to average over the distribution of tunes in the beam, namely,

$$\frac{1}{\tau_P(\nu)}=\int_{-\infty}^{\infty}\frac{1}{\tau_P(\nu')}G(\nu-\nu')d\nu', \quad (4)$$

where ν is the mean tune, and G is the tune distribution function.

C. Effect of an internal target

An example of a mixing mechanism is the interaction of beam particles with an internal target. Assume that a proton scatters from a target nucleus by an angle $\Delta\theta_y$ in the vertical plane. In the process, it changes its vertical betatron amplitude, and consequently the relevant resonance strength by an amount $\Delta\Gamma$ which is proportional to $\Delta\theta_y$. This causes a change in the angle α of the spin closed orbit, analogous to the situation described in the preceding section. It is easy to see that this leads to a depolarization rate that is proportional to $\xi\langle\Delta\theta_y^2\rangle$. However, since now the driving mechanism is known, the frequency and mean square of the scattering angle can be calculated as follows. At energies below 1 GeV Rutherford scattering is the dominant beam-target interaction. We can then integrate the square of the scattering angle, projected into the vertical plane, weighted with the Rutherford cross section between $\theta_{\rm min}$ and $\theta_{\rm max}$. The lower limit is given by Coulomb screening and the upper limit by the machine acceptance and the β function at the target. Since Rutherford scattering by angles larger than $\theta_{\rm max}$ is also responsible for the loss of protons from the ring [4], the rate of small-angle scattering can be related to the beam lifetime $\tau_{\rm beam}$. Detailed treatment shows that $\xi\langle\Delta\theta_y^2\rangle$ is proportional to $1/\tau_{\rm beam}$. The value for the constant C_Γ in Eq. (3), calculated along these lines, is several orders of magnitude larger than the experimentally determined value (see below), however, the calculation is uncertain because the distribution of the residual gas in the ring is not known. We will therefore base further arguments (Sec. IV B) on the conclusion that *if* scattering from the target and the rest gas were the dominating depolarization mechanism, the polarization lifetime τ_P would be proportional to the beam lifetime $\tau_{\rm beam}$.

III. EXPERIMENTAL DETAILS

A. The beam

A polarized 198 MeV proton beam is stored in the Cooler by kick injection [5]. The injected beam polarization is vertical, pointing either up (+) or down (−). Measurements have been carried out with both signs, as a check for systematic uncertainties. The accumulated beam current is typically 100–150 μA. The lifetime of the beam ranges from 1600 to 2200 s.

For the measurement of the mean tune, the beam is kicked, exciting a coherent, small-amplitude betatron oscillation. The tune is then deduced from the Fourier transform of the beam position, measured on a turn-by-turn basis [6]. During injection, the vertical tune was $\nu_y=4.868$, well separated from the intrinsic resonance tune which at this energy is $\nu_{yR}=4.827$ (see Sec. IV A).

B. The polarimeter

The beam polarization (more precisely, its vertical component) is determined by observing proton-proton elastic scattering from a polarized internal target, consisting of a thin-walled storage cell which is injected with a beam of polarized atoms [7,8]. A horizontal guide field, perpendicular to the beam, generated by Helmholtz coils outside the vacuum chamber, produces a target polarization in the desired direction. Reversing the guide field changes the sign of the target polarization in less than 50 ms. The target polarization is typically $P_{\mathrm{targ}}=0.78$, and stable over periods of several days [8]. The target, with a thickness of about 1.5×10^{13} atoms/cm^2, has a negligible effect on the beam lifetime.

Since the detector arrangement is described in detail in Ref. [8], we mention here only its main features. One of the outgoing protons is detected by a set of wire chambers and scintillator arrays, while the associated recoil proton is observed by an array of semiconductor detectors, mounted close to the target. The detection of both protons in coincidence provides a clean signature for pp scattering. The detector system covers scattering angles from 5° to 43° in the laboratory. For this angle range, yields are measured in four azimuthal directions, arranged symmetrically around the beam direction, namely, upper left (UL), lower left (LL), lower right (LR), and upper right (UR). Combining these yields as $G=(\mathrm{UL})+(\mathrm{LR})$, and $H=(\mathrm{LL})+(\mathrm{UR})$, the beam polarization P follows [8] from

$$P=\frac{2}{P_{\mathrm{targ}}(A_{xx}-A_{yy})}\,\frac{r-1}{r+1},\quad \text{where } r=\left(\frac{G_{<}H_{>}}{G_{>}H_{<}}\right)^{1/2}. \tag{5}$$

Here, P_{targ} and the subscripts $<,>$ refer to the magnitude and direction (left or right) of the target polarization, and A_{xx} and A_{yy} are the spin correlation coefficients in pp scattering which have been measured previously [8]. The large value of $A_{xx}-A_{yy}$, averaged over the covered angle range ($A_{xx}-A_{yy}=-1.475$), is a distinct advantage of this method to measure the beam polarization. An absolute calibration of the polarimeter is not necessary here, since we are only interested in the relative change $\Delta P/P$ with time.

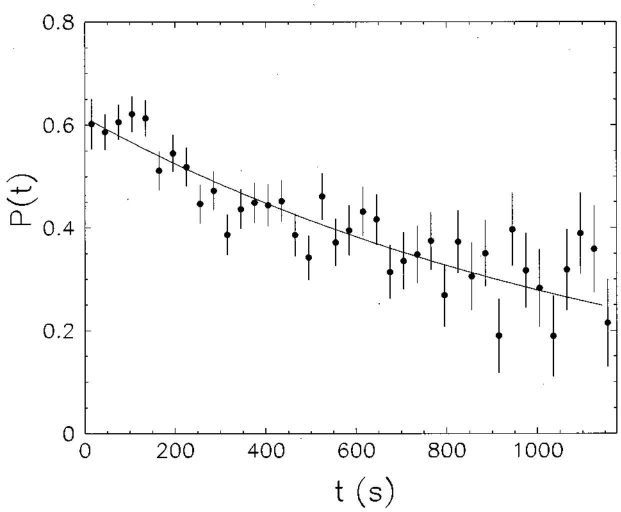

FIG. 2. Beam polarization as a function of time measured for a tune ν_y of 4.840. The line is an exponential fit to the data.

IV. MEASUREMENTS

A. Polarization lifetime near the intrinsic resonance

After injection, the beam is cooled for a few seconds, and the measurement of the polarization lifetime commences (see Sec. III B). An experimental cycle consists of injection of polarized beam into the ring, a change of machine tune to the desired value, followed by a data taking phase of 300–1200 s duration. During the measurement the target polarization is reversed every 2 s. For each 30 s interval the beam polarization is evaluated, using Eq. (5). This yields a measurement of the beam polarization $P(t_i)$ as a function of time, as shown in Fig. 2. These data are fitted with a two-parameter function $P(t)=P_0\exp(-t/\tau_P)$ where τ_P is the polarization lifetime. The error $\delta\tau_P$ is determined from the change in χ^2 obtained when changing τ_P, while still optimizing P_0.

The data presented here have been obtained over a two-day period in about 40 individual runs at 12 different settings of the quadrupole combination that adjusts the vertical tune. All runs at the same vertical tune ν_y were combined. The resulting polarization lifetime τ_P as a function of ν_y is shown in Fig. 3. Data have been obtained on one side of the resonance only, since shifting the tune through the resonance would destroy the polarization, and starting out at the upper side would have required a different machine setup.

This experiment is concerned with the polarization lifetime near the intrinsic resonance for which $G\gamma+\Delta_3=7-\nu_{yR}$. At 198 MeV, $G\gamma$ equals 2.171. The term Δ_3 represents a shift of the spin tune due to a ‘‘type-3 snake’’ [9], caused by the magnetic fields in the cooling region which introduce a precession around the vertical axis without a change in beam direction. Extrapolating the result of Ref. [10] to 197 MeV, we expect $\Delta_3=+0.002$, leading to a prediction for the resonance tune of $\nu_{yR}=4.827$.

The strength of this intrinsic resonance in the IUCF Cooler has been determined from resonance-crossing studies as $\Gamma=2\times10^{-4}$ for a normalized emittance of $0.25\pi\times10^{-6}$ m [11]. Since the emittance of a well-cooled beam is probably less than that, the resonance strength for this experiment could be somewhat smaller than the quoted number. However, since $\Gamma^2\ll(\nu_y-\nu_{yR})^2$ for all our measure-

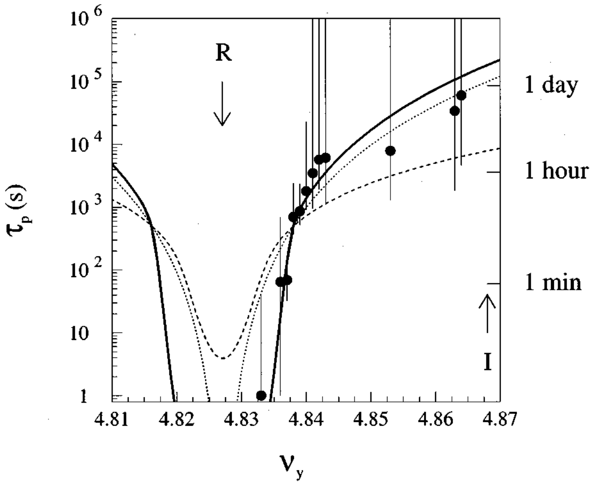

FIG. 3. Beam polarization lifetime as a function of the vertical tune ν_y. The dotted line represents Eq. (2) (the model which is based on the mixing of particle tunes). The solid line is the same, but folded [Eq. (4)] with an assumed Gaussian tune distribution with $\sigma_\nu = 0.002$. The dashed line represents Eq. (3) (the model which is based on the mixing of betatron amplitudes), folded with the tune distribution. The parameters used are $\nu_{yR} = 4.827$ (arrow ''*R*''), and $\Gamma = 2 \times 10^{-4}$, while C_δ and C_Γ are chosen arbitrarily. The tune at injection is 4.868 (arrow ''*I*'').

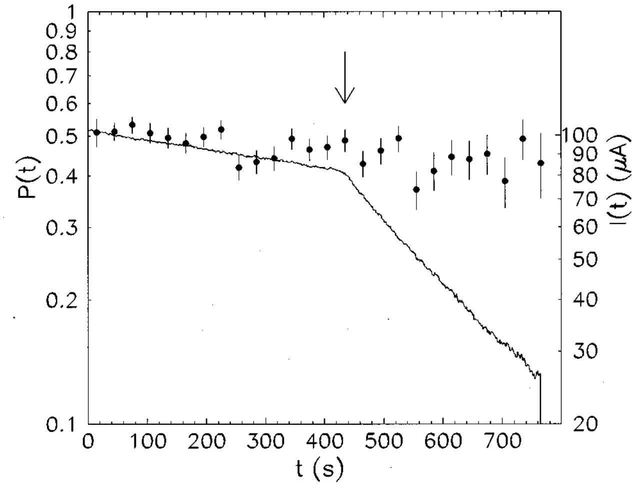

FIG. 4. Beam current (solid line, right-hand scale) and polarization (points, left-hand scale) as a function of time during a run for which $\nu_y = 4.842$. The arrow indicates the time at which the additional nitrogen target was turned on.

ments, this would not significantly affect Eq. (3) and only the normalization constant in Eq. (2).

The origin of the tune spread is not understood well. One possible candidate would be the defocusing effect of the space charge of the beam which makes the tunes of individual particles dependent on their betatron amplitude. This effect would also shift (decrease) the *actual* tune relative to the *measured* value by an amount that depends on the beam current. However, there is no evidence that supports this hypothesis, since we find no correlation between beam current and τ_P for otherwise identical conditions, and since previous studies [12,13] found the intrinsic resonance where it is supposed to be according to the measured tune. In the following we simply assume a Gaussian tune distribution.

The dotted line in Fig. 3 represents the model which is based on the mixing of particle tunes [Eq. (2)]. The agreement with the measurement is improved if this calculation is folded [Eq. (4)] with a Gaussian tune distribution with a variance of $\sigma_\nu = 0.002$ (solid line). The model which is based on the mixing of betatron amplitudes [Eq. (3)] is shown as a dashed line (also folded with the tune distribution). There is a slight preference of the tune-related mechanism over the one that involves the betatron amplitudes. In all of these calculations, the normalizations C_δ and C_Γ are chosen arbitrarily, such that $\tau_P = 500$ s at $\nu_y = 4.838$.

B. The effect of an internal target on the polarization lifetime

For some of the measurements, the beam lifetime τ_{beam} has been deliberately reduced by introducing a nitrogen gas target in the ''*T* region'' (about a third of the ring circumference upstream from the location of the polarimeter). The thickness of the N_2 target is typically 10^{14} N_2/cm^2 resulting in a reduction of the beam lifetime by about a factor of 4. When the N_2 target is used, it is turned on roughly halfway through the data taking period, such that the polarization lifetime is measured with and without N_2 target during the *same* run.

Figure 4 shows the time dependence of the beam current and the polarization for a typical run ($\nu_y = 4.842$). The arrow indicates the time when the nitrogen target was turned on. Clearly, this causes a change in the beam lifetime but there is no accompanying change in the polarization lifetime, as would be required by the target effect, discussed in Sec. II C. A compilation of all results of this kind is given in Fig. 5. Here, the polarization lifetimes with the N_2 target, τ_P^1, are shown versus the corresponding values τ_P^0, measured before the target was turned on. The errors shown are statistical.

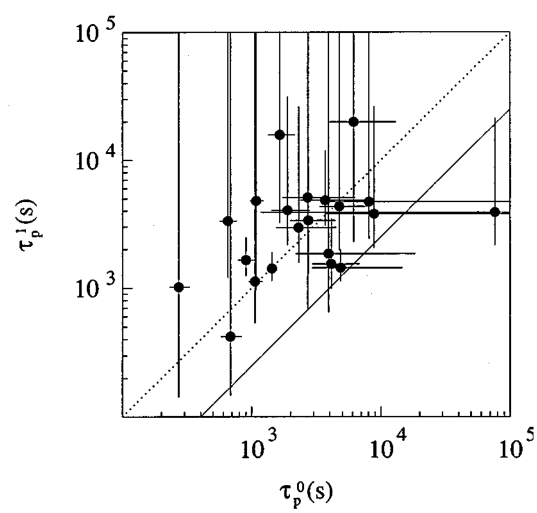

FIG. 5. Comparison of the polarization lifetimes τ_P^1 and τ_P^0 with and without an additional nitrogen target. The N_2 target lowers the beam lifetime by about a factor of 4. The data are expected to lie on the solid line if the polarization lifetime were proportional to the beam lifetime, as required by the mechanism discussed in Sec. II C. The dashed line shows the locus which is expected if there is no effect of the target on the polarization lifetime.

The data are expected to lie on the solid line if the polarization lifetime were proportional to the beam lifetime, as required by the mechanism discussed in Sec. II C, and on the dashed line if there is no effect of the target on the polarization lifetime. From Figs. 4 and 5 it is clear that depolarization by scattering from internal targets is negligible compared to other depolarization mechanisms.

V. SUMMARY, DISCUSSION

We have measured the lifetime of the polarization of a stored proton beam near an intrinsic depolarizing resonance as a function of the distance from the resonance. To measure the beam polarization, an internal, polarized target was used. This polarimeter has a very large effective analyzing power, and, due to its low density, it has a negligible effect on the stored beam lifetime.

To explain depolarization, phase-space mixing is needed. The nonconservative mechanism responsible for this mixing is unknown. The measured polarization lifetimes, up to a scale factor, are explained by a model that assumes some form of mixing of individual-particle betatron tunes. Our data rule out a previously proposed mechanism that is based on the mixing of betatron amplitudes driven by the scattering from internal targets (including the rest gas).

The dependence of the polarization lifetime $\tau_P(\delta)$ on the distance from the resonance, given by Eq. (2), has an interesting consequence: it means that it is usually not possible to deduce the width of the resonance (which is equivalent to the strength Γ) from a polarization measurement. This is obvious because Γ contributes significantly only if $\delta \sim \Gamma$, while the beam polarization decays quickly as the resonance is approached even while δ is still much larger than Γ.

New theoretical ideas for mixing mechanisms can be tested, because they lead to a prediction of the normalizing constants in Eqs. (2) and (3). On the other hand, future experimental studies are also conceivable that do not rely on an available model, but might provide inspiration for one. For instance, it would be desirable to improve the present measurement, mapping out $\tau_P(\delta)$ with higher precision. Note, however, that measurements spanning a large range in τ_P are difficult because for short τ_P counting statistics are a problem, while for long τ_P extremely stable experimental conditions are mandatory.

Other future experiments could explore the role of nonconservative processes such as electron cooling (study of τ_P with and without cooling), intrabeam scattering (study of τ_P as a function of the stored number of particles), or synchrotron motion (study of τ_P with bunched and coasting beam). It also would be interesting to repeat the present experiment near an imperfection resonance (study of τ_P as a function of the beam energy).

The most promising τ_P study near a resonance, however, would make use of a ''rf-induced'' depolarizing resonance [14]. Such a resonance is generated by a longitudinal magnetic field, alternating at a frequency given by the orbit frequency times the spin tune. This resonance can be turned on and off with ease, its strength is known and can be varied, and the distance from the resonance can be adjusted at will without changing any of the machine parameters. Such an investigation would be complementary to the present study because, for an induced resonance, the resonance condition, to first order, does not depend on the beam tune nor the transverse phase space. Since the induced resonance can be turned on for an arbitrarily short time, while measuring polarization before and after, it would be possible to investigate the (short) polarization lifetime *exactly on resonance*. This would provide important insight into the efficiency of flipping the spin of a stored beam by crossing a rf-induced resonance [15,16].

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